BMED436 Medical Robot
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Forward Kinematics

1 Exercise 3.3

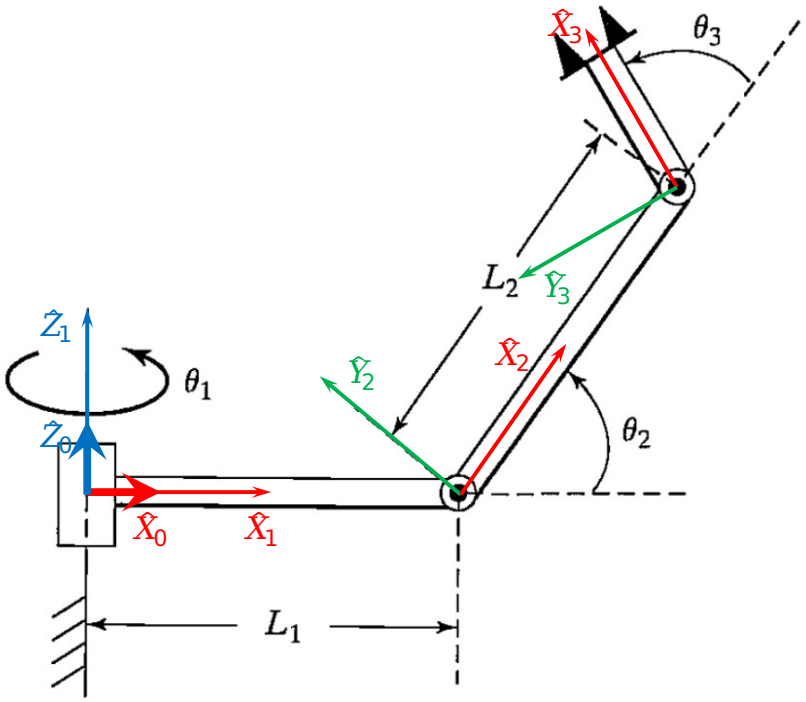

Figure 1: RRR planar arm. Link frames are assigned.

Figure 1 shows the link frames assigned for an RRR planar arm. Listing 1 shows the MATLAB code where the transform matrices are computed using DH parameters from Table 1. Refer to function 'link_transform' in Listing 3.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	L_1	0	θ_2
3	0	L_2	0	θ_3

Table 1: DH Parameters for RRR planar arm.

```
syms L1 L2 theta1 theta2 theta3

DH = [
    0 0 0 theta1;
    pi/2 L1 0 theta2;
    0 L2 0 theta3
    ];

link_transform(DH);
```

Listing 1: Transformation matrix computation for RRR planar arm.

Transformation Matrix from Frame 0 to Frame 1:

$$\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrix from Frame 1 to Frame 2:

$$\begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrix from Frame 2 to Frame 3:

$$\begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrix from Frame 0 to Frame 3:

$$\begin{bmatrix} \cos(\theta_2+\theta_3)\cos(\theta_1) & -\sin(\theta_2+\theta_3)\cos(\theta_1) & \sin(\theta_1) & \cos(\theta_1)(L_1+L_2\cos(\theta_2)) \\ \cos(\theta_2+\theta_3)\sin(\theta_1) & -\sin(\theta_2+\theta_3)\sin(\theta_1) & -\cos(\theta_1) & \sin(\theta_1)(L_1+L_2\cos(\theta_2)) \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & L_2\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 Exercise 3.16

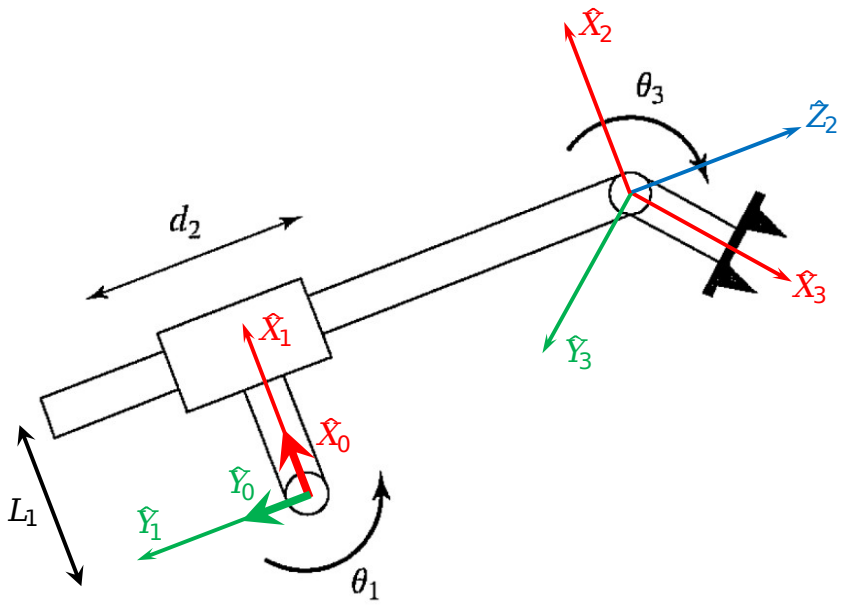

Figure 2: RPR planar robot. Link frames are assigned.

Figure 2 shows the link frames assigned for an RPR planar robot. Listing 2 shows the MATLAB code where the transform matrices are computed using DH parameters from Table 2. Refer to function 'link_transform' in Listing 3.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	L_1	d_2	0
3	90°	0	0	θ_3

Table 2: DH Parameters for RPR planar robot.

```
syms L1 d2 theta1 theta3

DH = [
    0 0 0 theta1;
    pi/2 L1 d2 0;
    pi/2 0 0 theta3
    ];

link_transform(DH);
```

Listing 2: Transformation matrix computation for RPR planar robot.

Transformation Matrix from Frame 0 to Frame 1:

$$\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrix from Frame 1 to Frame 2:

$$\begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrix from Frame 2 to Frame 3:

$$\begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrix from Frame 0 to Frame 3:

$$\begin{bmatrix} \cos(\theta_1 - \theta_3) & \sin(\theta_1 - \theta_3) & 0 & L_1\cos(\theta_1) + d_2\sin(\theta_1) \\ \sin(\theta_1 - \theta_3) & -\cos(\theta_1 - \theta_3) & 0 & L_1\sin(\theta_1) - d_2\cos(\theta_1) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Supplementary Material

```
function link_transform(DH)
    T = eye(4);
    for i = 1:size(DH,1)
        screw_X = [
            1 0 0 DH(i,2);
            0 cos(DH(i,1)) -sin(DH(i,1)) 0;
            0 sin(DH(i,1)) cos(DH(i,1)) 0;
            0 0 0 1
            ];

        screw_Z = [
            cos(DH(i,4)) -sin(DH(i,4)) 0 0;
            sin(DH(i,4)) cos(DH(i,4)) 0 0;
            0 0 1 DH(i,3);
            0 0 0 1
            ];

        T_intermediate = screw_X * screw_Z;
        disp("T from Frame " + (i-1) + " to Frame " + i + ".");
        disp(simplify(T_intermediate));

        T = T * T_intermediate;
    end

    disp("T from Frame 0 to Frame " + size(DH,1) + ".");
    disp(simplify(T));
```

Listing 3: Computes transformation matrices using DH parameters.